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Pension Funds as Institutions for Intertemporal Risk Transfer

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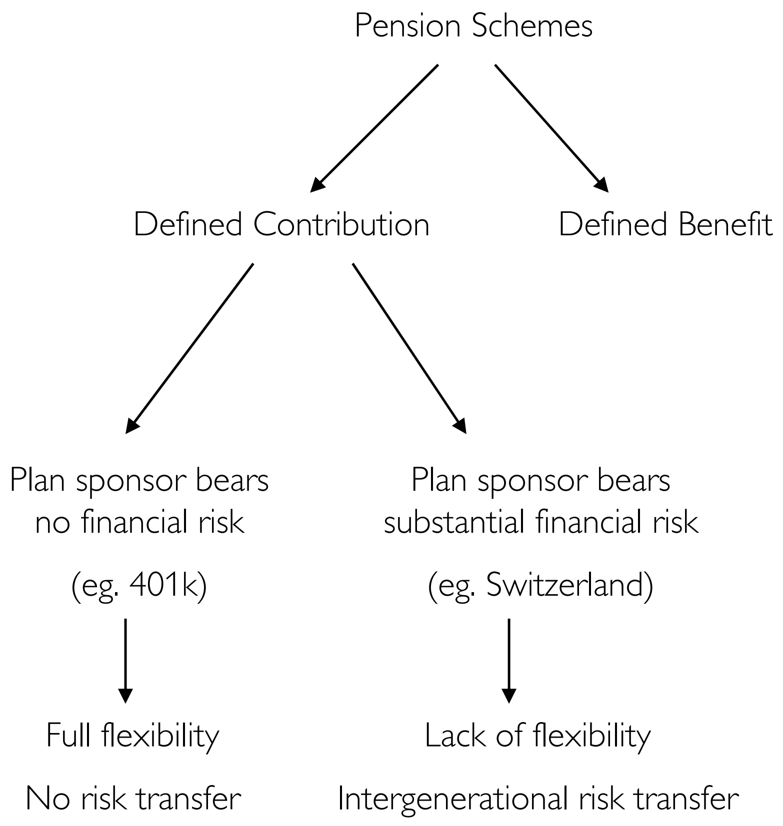
Pension Schemes



Pension Schemes

Defined Contribution

Defined Benefit

Plan sponsor bears
no financial risk

(eg. 401k)

Full flexibility

No risk transfer

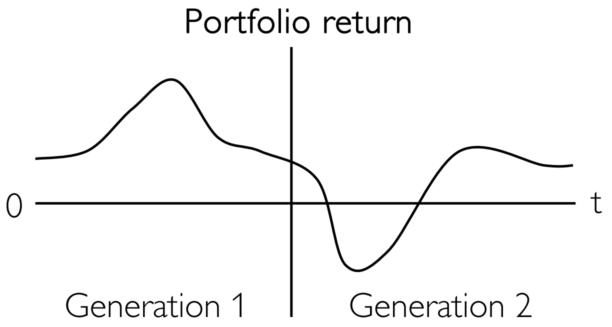
Plan sponsor bears
substantial financial risk

(eg. Switzerland)

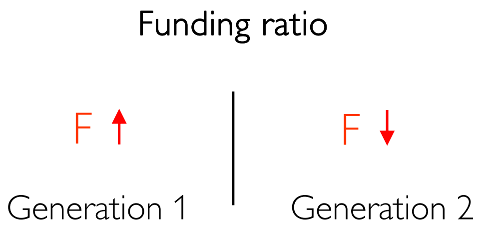
Lack of flexibility

Intergenerational risk transfer

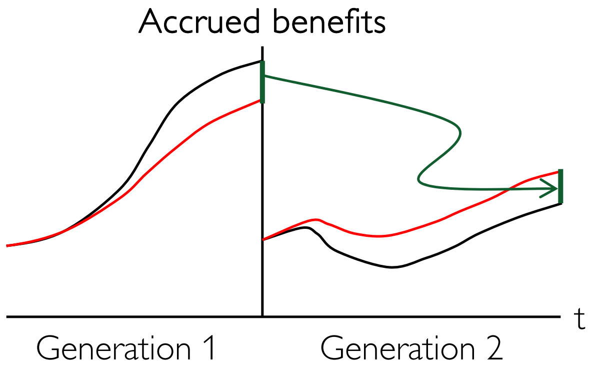
Intergenerational Risk Transfer



The returns are high during the time of employees-generation 1 and low during the time of employees-generation 2.



The funding ratio serves as an instrument for redistribution of financial risk.



Generation 1 profits from the good markets, generation 2 suffers from the bad markets.

A part of the accrued benefits of generation 1 is transferred to the funding ratio. This reserves are used to increase the accrued benefits of generation 2.

There is a transfer from generation 1 to generation 2.

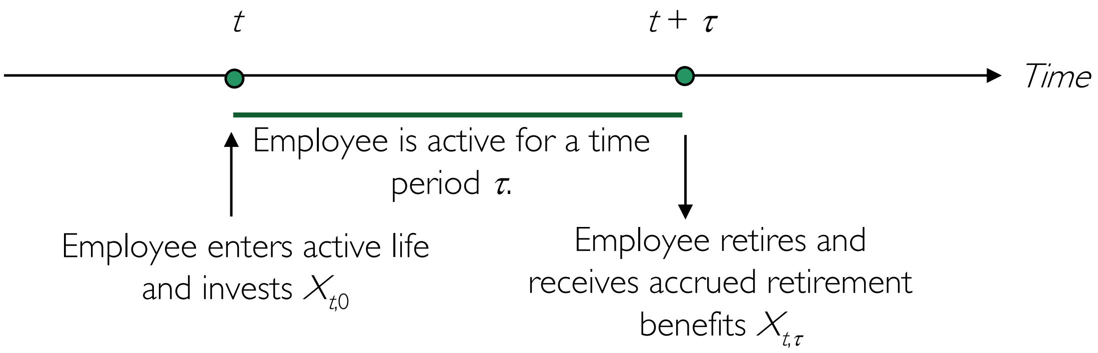
Intergenerational Risk Transfer

A plan-policy which increases the funding ratio if markets perform well and decreases it in case of poor market performance leads to an intergenerational transfer of financial risk.

It becomes possible to diversify intertemporally.

Such risk transfers were analyzed in a general context by Allen and Gale (1997).

Model Basics



t

$t+\tau$

Time

Employee is active for a time period τ.

Employee enters active life and invests $X_{t,0}$

Employee retires and receives accrued retirement benefits $X_{t,\tau}$

Employees **utility** CRRA: $u(x) = (1-R)^{-1} x^{1-R} \quad , R>1$

The **price processes** of N assets are given by geometric Brownian motions:

$$\frac{dS_{0,t}}{S_{0,t}} = rdt \qquad \left(\frac{dS_{i,t}}{S_{i,t}}\right)_{i=1,\dots,N} = (r + \pi_{i=1,\dots,N})dt + \sigma dZ_t$$

risk free rate

risk premia of asset i

regular matrix

N-dimensional standard Brownian Motion

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Autonomous Case

Our simplified model for a defined-contribution plan in the narrow sense leads to the famous **Merton solution** in case that every employee acts autonomously.

$$x^{M} = \frac{1}{R}\left(\sigma\sigma^{\top}\right)^{-1}\pi$$

relative risk aversion

risk premiums

(inverse) covariance matrix

Risk Transfer Case: Agenda

- Risk transfer model
- Optimal fund strategy for each generation (dynamic and static)
- Implicit risk tolerance for different generations
- Welfare optimizing investment strategy
- Pareto improvement due to intergenerational risk transfer
- Analysis of the funding ratio

static investment strategy

Risk Transfer Case: Model

The intergenerational risk transfer is implemented by a fund, which is modelled as follows:

A_t : value of assets at time t
L_t : value of liabilities at time t
$F_t = A_t / L_t$: funding ratio at time t

Stochastic process of the **assets**: $dA_t = A_t(r + \mathbf{x}_t^\top \pi)dt + A_t \mathbf{x}_t^\top \sigma d\mathbf{Z}_t + C_t dt$

where C_t denotes the net contributions to the fund.

Participation:

The fund attributes a return $r + k\ln\left(\frac{F_t}{\bar{F}}\right)$ to the accrued retirement benefit accounts of employees.

- uniquely depending on the funding ratio F_t,
- below the **critical level** $\bar{F}$, employees get less than the riskfree rate.
- **high and low asset returns of the fund are spread over future dates**
- **intergenerational risk transfer** takes place.

Accrued benefits of an employee who entered active life in t: $dX_{t,s} = X_{t,s}\left(r + k\ln\left(\frac{F_{t+s}}{\bar{F}}\right)\right)dt$

Stochastic process of the **liabilities**: $dL_t = L_t\left(r + k\ln\left(\frac{F_t}{\bar{F}}\right)\right)dt + C_t dt$

Risk Transfer Case: Further Assumption

Assumption: No net contributions!

$$C_t = 0$$

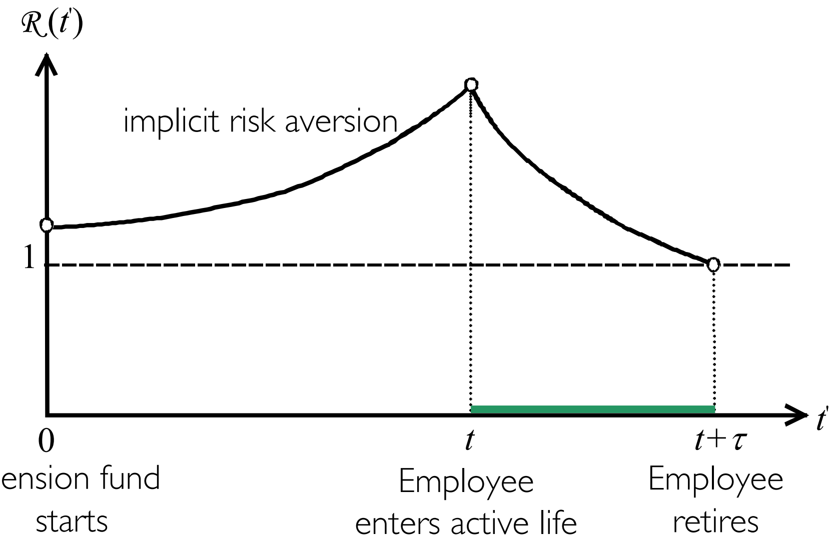
It suggests a growing number of members in the pension fund.

Optimal Dynamic Fund Strategy for Each Generation

Proposition 1:

The investment policy $\mathrm{x}_{t'}$, $0 \leq t' \leq t+\tau$, maximizing $\mathrm{E}_0\left[(1-R)^{-1} X_{t,\tau}^{1-R}\right]$ is given by

$$\mathrm{x}_{t'} = \frac{1}{\Re(t')}\left(\sigma\sigma^{\mathrm{T}}\right)^{-1}\pi \qquad \text{with} \qquad \Re(t') = \begin{cases} 1+(R-1)\left(1-e^{-k\tau}\right)e^{k(t'-t)} & , 0 < t' \leq t \\ 1+(R-1)\left(1-e^{k(t'-t-\tau)}\right) & , t < t' \leq t+\tau \end{cases}$$



Comments:

The investment policy becomes more conservative until the employee enters active life in t.

Afterwards it becomes more aggressive and at retirement in $t+\tau$ the growth optimum portfolio is attained.

For all $t' \in [0, t+\tau]$ the investment policy $\mathrm{x}_{t'}$ is more aggressive than the Merton portfolio x^{M}.

Optimal Static Fund Strategy for Each Generation

Proposition 2:

1) The optimal static portfolio for an employee entering active life in t is $\mathbf{x}(t) = \frac{1}{R}\frac{c(t,k,\tau)}{d(t,k,\tau)}\left(\sigma\sigma^{\top}\right)^{-1}\pi$

with $c(t,k,\tau) = \tau - \frac{1-e^{-k\tau}}{k}e^{-kt}$

$$d(t,k,\tau) = \tau - \frac{1-e^{-k\tau}}{k}\left[1+\tfrac{1}{2}e^{-2kt}\left(1-e^{-k\tau}\right)\right] + \frac{1}{R}\frac{1-e^{-k\tau}}{k}\left[1-e^{-kt}+\tfrac{1}{2}e^{-2kt}\left(1-e^{-k\tau}\right)\right]$$

2) $\frac{1}{R} < \frac{c(t,k,\tau)}{Rd(t,k,\tau)} < 1, \quad d(t,k,\tau) > 0 \quad$ for all t, k, τ

Comments:

Thus, in the risk transfer model all employees prefer portfolios $\mathbf{x}(t)$ which are more aggressive than the Merton solution $\mathbf{x}^{M}$, but less aggressive than the growth optimum portfolio.

x(t) does not depend on F_0 , F_t or $\bar{F}$.

Risk Tolerance For Different Generations

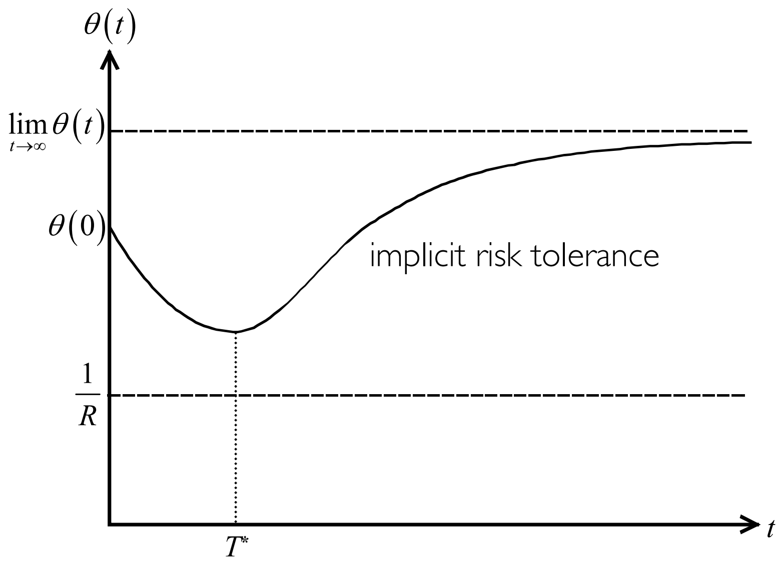
Define $\theta(t) = \frac{1}{R}\frac{c(t,k,\tau)}{d(t,k,\tau)}$ (implicit risk tolerance) We already know that $\frac{1}{R} < \theta(t) < 1$

Proposition 3:

1) $\theta(0) < \lim_{t\to\infty} \theta(t) < 1$

2) There exists $T^* > 0$ not depending on R such that

$\theta(t)$ is strictly decreasing for $0 < t < T^*$
$\theta(t)$ is strictly increasing for $t > T^*$.



Comments:

Employees entering active life in the near future suffer from the risk with respect to the future funding ratio.

For employees entering in the far distant future, expected return plays a more important role.

The optimal portfolio $x(t)$ for an employee entering active life at time t with

$x(t) = \lambda\left(\sigma\sigma^{\top}\right)^{-1}\pi$ and $\theta\left(T^*\right) \le \lambda < \lim_{t\to\infty}\theta(t)$

is Pareto efficient.

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Welfare Optimizing Static Investment Strategies

We use the **welfare function** as $\int_0^\infty e^{-\delta t}\left\{E_0\left[\ln\left(\frac{X_{t,\tau}}{X_{t,0}}\right)\right]+\frac{1-R}{2}Var_0\left[\ln\left(\frac{X_{t,\tau}}{X_{t,0}}\right)\right]\right\}dt$

Proposition 4:

1) The welfare maximizing portfolio is given by $\mathsf{x}=\frac{1}{R}\frac{c(\delta,k,\tau)}{d(\delta,k,\tau)}\left(\sigma\sigma^{\top}\right)^{-1}\pi$

with $c(\delta,k,\tau)=\frac{\tau}{\delta}-\frac{1-e^{-k\tau}}{k}\frac{1}{k+\delta}$

$$d(\delta,k,\tau)=\frac{\tau}{\delta}-\frac{1-e^{-k\tau}}{k}\left\{\left(1-\frac{1}{R}\right)\left[\frac{1}{\delta}+\frac{1}{2}\frac{1}{2k+\delta}\left(1-e^{-k\tau}\right)\right]+\frac{1}{R}\frac{1}{k+\delta}\right\}$$

2) $\frac{1}{R}<\frac{c(\delta,k,\tau)}{Rd(\delta,k,\tau)}<1$, $d(\delta,k,\tau)>0$ for all δ, k, τ

Comments:

The welfare maximizing portfolio is more aggressive than the Merton portfolio x^{M}, but less aggressive than the growth optimum portfolio.

Probably, the welfare maximizing portfolio is not a Pareto improvement relative to the Merton solution x^{M}.

Intergenerational Risk Transfer: Pareto Improvement

Proposition 5:

Assume $F_0 = 1$. Then for $x^* = \lambda^* (\sigma\,\sigma^{\top})^{-1}\pi$ with $\lambda^* = \min_{t \geq 0} \frac{1}{R} \frac{c(t,k,\tau)}{d(t,k,\tau)}$

there exists $\bar{F} < 1$ such that

1) In comparison with the autonomous case all employees attain a higher anticipated expected utility.
2) $\mathrm{med}(F_t) > 1$ for all $t > 0$.

Comments:

It is quite natural to assume that the plan starts fully funded at t=0, i.e. $F_0 = 1$.

The property $\mathrm{med}(F_t) > 1$ is a condition for the financial stability of the pension plan.

Under the portfolio x^* and $\bar{F} < 1$ all employees are better off than in the autonomous case.

By taking advantage of the increase in risk tolerance a Pareto improvement can be achieved under the stability condition $\mathrm{med}(F_t) > 1$.

Analysis Of The Funding Ratio

The **α-percentiles** of $Y_t = \ln(F_t)$: $p(t, \alpha) = E(Y_t) + z_\alpha \sigma(Y_t)$

Proposition 6:

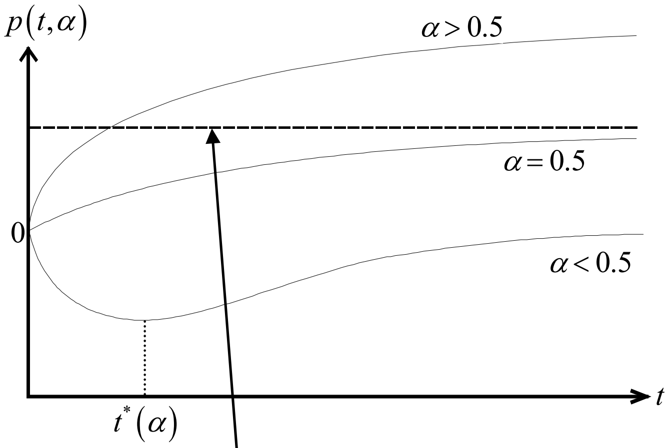
1) $p(0, \alpha) = 0$

2) $\lim_{t\to\infty} p(t,\alpha) = \ln(\bar{F}) + \frac{x^T\pi - \frac{1}{2}x^T\sigma\sigma^T x}{k} + z_\alpha \left(\frac{x^T\sigma\sigma^T x}{2k}\right)^{\frac{1}{2}}$

3) $p(t, \alpha)$ is strictly increasing and concave in t for $\alpha \geq 0.5$

4) If $\alpha < 0.5$, then $p(t, \alpha)$ attains a minimum at

$$t^*(\alpha) = \frac{1}{2k}\ln\left(1 + \frac{z_\alpha^2 x^T\sigma\sigma^T x}{2k\left(\ln(\bar{F}) + \frac{x^T\pi - \frac{1}{2}x^T\sigma\sigma^T x}{k}\right)^2}\right)$$



$\lim_{t\to\infty} med(Y_t) = \ln(\bar{F}) + \frac{x^T\pi - \frac{1}{2}x^T\sigma\sigma^T x}{k}$

Comments:

Employees entering active life in the very near or in the far distant future suffer less under the uncertainty with respect to the funding ratio than those entering in the intermediate future.

k should not be too large or too small.

In the long run the conditional median of the funding ratio F_T always tends to a value larger than 1.

Conclusions

- Intergenerational risk transfer leads to an increase in implicit risk tolerance for all future employees.
- All future employees can attain a higher anticipated expected utility. In this sense a Pareto improvement can be achieved.
- However, such a framework is inappropriate for an individual choice of the investment policy. Moreover, there is a risk of underfunding.
- At the starting point of the plan intergenerational risk transfer leads to an increase of the anticipated expected utility for all future members. However, later on the plan may get underfunded and unattractive for new members.
- A pension plan with intergenerational risk transfer should be supplemented by a plan with individual investment decisions and no transfer of financial risks.